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ON THE STABILITY OF RODS FOR STOCHASTIC EXCITATIONS*

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The stability of motion of an elastic rod in a viscous medium compressed by a randomly acting force is studied. The conditions of stability of the rod acted upon by a stationary process with bilinear spectral density are obtained. The dependence of the statistical moments of the amplitude of the finite flexure of the rod under stationary-motion conditions on the parameters of the compressing force and the amplitude of the initial deformation is analysed. A number of problems concerning the stability of longitudinal flexure of viscoelastic constructions acted upon by random loads were discussed in /1-3/.

1. A stationary process with rational-fraction spectral density. Let us consider an elastic rod of length l, hinged at each end and compressed by forces F. The rod is in a continuous viscous medium and its equation of equilibrium has the form

$$\partial w/\partial t = -A\,\{EIw^{\mathrm{IV}} + [F_0 + F_1(t)]\,w''\} \tag{1.1}$$

Here A is the viscosity constant of the material of the medium, F_0, $F_1(t)$ are the deterministic (constant with respect to time) component of the compressive load, and a random oscillation with zero expectation value. The remaining notation is the generally accepted one.

Let the deflection of the rod at the initial instant be described by the sinusoid

$$w(0, x) = f_k^\circ \sin(k\pi x/l)$$

We shall seek a solution of (1.1) in the form of such a sinusoid, whose amplitude $f_k(t)$ is a solution of the equation

$$df_k/d\tau + k^4\,[(1-\alpha_k) - \beta_k\psi(\tau)]\,f_k = 0 \tag{1.2}$$

$$\tau = \gamma t, \quad \gamma = \frac{\pi^4 EIA}{l^4}, \quad \alpha_k = \frac{F_0 l^2}{k^2\pi^2 EI}, \quad \beta_k\psi(\tau) = \frac{F_1(\tau)\,l^2}{k^2\pi^2 EI}$$

(β_k is a deterministic constant).

Let us assume that the random process $\psi(\tau)$ is the result of the passage of normal "white noise" through a linear filter

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$$d^i\psi/d\tau^i + a_1 d^{i-1}\psi/d\tau^{i-1} + \ldots + a_i\psi = \mu\xi \tag{1.3}$$

where $a_1, \ldots, a_i, \mu$ are constants.

In the quasistatic formulation the solution of the stochastically non-linear problem reduces to solving a system of two differential equations with initial conditions

$$f_k(0) = f_k^\circ, \quad \psi(0) = d\psi/d\tau\,|_{\tau=0} = \ldots = d^{i-1}\psi/d\tau^{i-1}\,|_{\tau=0} = 0$$

The unknown functions $f_k, \psi, d\psi/d\tau, \ldots, d^{i-1}\psi/d\tau^{i-1}$ determine the components of a multidimensional Markov process and their combined probability distribution density can be found from the Fokker-Kolmogorov (FPK) equation. It should be noted that this equation has the same form in the sense of Ito as well as of Stratonovich when white noise is considered.

Introducing the new variable $z_k = \ln f_k$, we can reduce the non-linear problem to a linear one, and

$$dz_k/d\tau = -k^4[(1-\alpha_k) - \beta_k\psi], \quad z_k(0) = \ln f_k^\circ \tag{1.4}$$

In order to determine the distribution density of the process z_k at the instant τ, it is sufficient to find its expectation value and variance.

Averaging the left and right-hand sides of (1.4) over the set of samples, we obtain the equation for the expectation value. Integrating this equation we obtain

$$\langle z_k\rangle = \ln f_k^\circ - k^4(1-\alpha_k)\tau \tag{1.5}$$

The random fluctuation of the function z_k is found from the relation

$$dz_k^*/d\tau = k^4\beta_k\psi, \quad z_k^* = z_k - \langle z_k\rangle \tag{1.6}$$

which yields, taking (1.3) into account,

$$\frac{d^{i+1}z_k^*}{d\tau^{i+1}} + a_1\frac{d^i z_k^*}{d\tau^i} + \ldots + a_i\frac{dz_k^*}{d\tau} = k^4\beta_k\mu\xi$$

whose solution is found using the Laplace transformation

$$z(p) = k^4\beta_k\mu\,(p\Delta(p))^{-1}F(p) \tag{1.7}$$

$$\Delta(p) = p^i + a_1p^{i-1} + \ldots + a_i$$

$$\left(z(p) = \int_0^\infty e^{-p\tau}z_k^*(\tau)\,d\tau, \qquad F(p) = \int_0^\infty e^{-p\tau}\xi(\tau)\,d\tau\right)$$

We shall impose one restriction on the constant coefficients of Eq.(1.3): we shall assume that the roots $p_1, p_2, \ldots, p_i$ of the characteristic equation $\Delta(p) = 0$ have negative real roots.

The expression $(p\Delta(p))^{-1}$ can be written in the form

$$\frac{1}{p\Delta(p)} = \frac{1}{a_ip} + \sum_{j=1}^{m}\sum_{\lambda=1}^{\nu_j} c_j^{(\lambda)}(p-p_j)^{-\lambda}, \quad \nu_1+\nu_2+\ldots+\nu_m = i \tag{1.8}$$

where ν_j is the multiplicity of the root p_j, $c_j^{(\lambda)}$ are constants which can be found by comparing the coefficients of like powers of p in the numerators of the left and right-hand sides of (1.8).

Taking into account (1.8) we obtain, from (1.7),

$$\langle z_k^{*2}(\tau)\rangle = k^8\beta_k^2\mu^2\left[\frac{\tau}{a_i^2} + \frac{2}{a_i}\int_0^\tau\Phi(\tau,\theta)\,d\theta + \int_0^\tau\Phi^2(\tau,\theta)\,d\theta\right] \tag{1.9}$$

$$\Phi(\tau,\theta) = \sum_{j=1}^{m}\sum_{\lambda=1}^{\nu_j-1}\frac{1}{(\lambda-1)!}c_j^{(\lambda)}\exp[p_j(\tau-\theta)](\tau-\theta)^{\lambda-1}$$

The probability distribution densities of the random functions $z_k(\tau)$, $f_k(\tau)$ are given by the expressions

$$v[z_k(\tau)] = [2\pi\langle z_k^{*2}(\tau)\rangle]^{-1/2}\exp\left\{-\frac{[z_k(\tau)-\langle z_k(\tau)\rangle]^2}{2\langle z_k^{*2}(\tau)\rangle}\right\}$$

$$v_1[f_k(\tau)] = [2\pi\langle z_k^{*2}(\tau)\rangle f_k^2(\tau)]^{-1/2}\exp\left\{-\frac{[\ln f_k(\tau)-\langle z_k(\tau)\rangle]^2}{2\langle z_k^{*2}(\tau)\rangle}\right\}$$

Having obtained these densities, we can find the n-th order moments of the flexure amplitude of the rod

$$\langle f_k^n(\tau)\rangle = \langle \exp[nz_k(\tau)]\rangle = \exp[n\langle z_k(\tau)\rangle + {}^1/_2 n^2 \langle z_k^{*2}(\tau)\rangle] \quad (1.10)$$

The integrals of the functions $\Phi(\tau, \theta)$, $\Phi^2(\tau, \theta)$, in (1.9) contain terms of the type

$$I_1 = \frac{1}{(\lambda-1)!} c_j^{(\lambda)} \int_0^\tau (\tau-\theta)^{\lambda-1} \exp[p_j(\tau-\theta)]\, d\theta$$

Since the real parts of the roots $p_1, p_2, \ldots, p_j$ are negative, it follows that the quantities I_1 will tend, as τ increases, to constant values depending on $c_j^{(\lambda)}$, p_j and λ.

Thus, beginning from some fairly long instant of time τ, the following relations will hold for the moments $\langle f_k^n(\tau)\rangle$:

$$\langle f_k^n(\tau)\rangle \leqslant f_k^{\circ n} \exp[-nk^4(1-\alpha_k)\tau + {}^1/_2 n^2 k^8 \beta_k^2 \mu^2 a_i^{-2} \tau + \mathrm{const}]$$

This implies that when the time increases without limit, the static moments of the flexure amplitude of the rod will tend to zero, provided that the following conditions hold:

$$\alpha_k < 1 - {}^1/_2 n\mu^2 a_i^{-2} k^4 \beta_k^2 = 1 - {}^1/_2 n\mu^2 a_i^{-2} \beta_1^2 \quad (1.11)$$

Thus the inequalities (1.11) represent the criterion of asymptotic n-stability /4/ of the motion of the rod (also asymptotically stable with respect to the n-th order moments of the solution of (1.2)). Since $\alpha_k < \alpha_1$ when $k > 1$, it is clear that conditions (1.11) will hold for any k, provided it holds for $k = 1$.

This leads us to the following conclusion. If the motion of the rod is asymptotically n-stable in the case when the initial distortion of its axis can be described by a single sinusoidal half-wave, it will also be asymptotically stable in the case when its initial flexure is described by a single sinusoid with a large number of half-waves. We note that in the course of analysing the stability of an elastic rod in the deterministic formulation, the critical Euler force is also determined from the condition that the distortion of the rod axis at the instant of bifurcation is described by a single sinusoidal half-wave.

We shall give, for comparison, the condition of stability obtained in /1/ under the assumption that the process $\psi(\tau)$ represents white noise $\xi(\tau)$ in the Stratonovich sense. In this case the condition of asymptotic stability of the trivial solution of Eq.(1.2) with respect to the n-th order moments was found to be as follows: $\alpha_k < {}^1/_2 n\beta_1^2$. The condition of asymptotic decay of the moment $\langle f_k^n(\tau)\rangle$ (1.11) in the case of the random process $\psi(\tau)$, with bilinear spectral density, is found to be exactly the same as in the case when we assume that the white noise intensity coefficient is equal to μ/a_i. Therefore, the condition of stability of the slow motion of an elastic rod in a viscous medium can be obtained by considering not a system of $i+1$ stochastic first-order differential equations, but a single (simpler) equation.

Example. Let us assume that the random process $\psi(\tau)$ is the result of the passage of normal white noise through a linear first-order filter

$$d\psi/d\tau = -\eta\psi + \mu\xi, \quad \psi(0) = 0; \quad \eta,\ \mu = \mathrm{const} \quad (1.12)$$

From (1.6) and (1.12) we obtain

$$\langle z_k^{*2}(\tau)\rangle = k^8 \beta_k^2 \mu^2 \eta^{-2} [\tau - 2\eta^{-1}(1 - e^{-\eta\tau}) + {}^1/_2 \eta^{-1}(1 - e^{-2\eta\tau})]$$

The moments of the flexure amplitude of the rod are

$$\langle f_k^n(\tau)\rangle = \{f_k^\circ \exp[(-1 + \alpha_k + {}^1/_2 n\mu^2\eta^{-2}\beta_1^2) k^4\tau + n\mu^2\eta^{-3}k^8\beta_k^2(-{}^3/_4 + e^{-\eta\tau} - {}^1/_4 e^{-2\eta\tau})]\}^n$$

It is clear that when the time increases without limit, the moments will tend to zero provided that condition (1.11) holds when $a_i = \eta$.

2. Dynamic formulation of the problem. In order to assess the effect of inertial forces on the stability of the rod acted upon by a longitudinal force in the form of a stationary random process, we shall study once again the same rod whose equation of motion differs from (1.1) by the additional term $-Amd^2w/dt^2$ on the right-hand side, where m is the mass per unit length of the rod, constant along its length.

Assuming that the distortion of the rod at the initial and current instants of time can be represented in the form of a single sinusoid, we obtain the following equation for the amplitude of the flexure:

$$d^2f_k/d\tau^2 + \Omega^2 df_k/d\tau + k^4\Omega^2[1 - \alpha_k - \beta_k\psi(\tau)] f_k = 0, \quad \Omega^2 = \pi^2 EI/(ml^4\gamma^2) \quad (2.1)$$

Let us assume that the function $\psi(\tau)$ is normal white noise $\xi(\tau)$.

The theory of stability of stochastic systems contains a theorem /4/, according to which the necessary and sufficient condition for the mean square asymptotic stability of the solution of Eq.(2.1) is that conditions $\Delta_2 > 0$ and $\Delta_2 > \Delta/2$ hold, where

$$\Delta_2 = \begin{vmatrix} \Omega^2 & 0 \\ 1 & k^4\Omega^2(1-\alpha_k) \end{vmatrix}, \quad \Delta = \begin{vmatrix} 0 & -k^4\Omega^4\beta_1^2 \\ 1 & \Omega^2(1-\alpha_k) \end{vmatrix}$$

From this it follows that

$$\alpha_k < 1 \text{ and } \alpha_k < 1 - {}^1/_2\beta_1^2$$

The second of these conditions is identical with the condition of mean square asymptotic stability of the rod in the quasistatic formulation of the problem /1/.

The sufficient conditions were also formulated in /4/ for the asymptotic stability of the solution of (2.1) with respect to the statistical n-th order moments $\Delta_2 > 0$ and $\Delta_2 > {}^1/_2(n-1)\Delta$, for which we have the following corresponding relations:

$$\alpha_k < 1, \ \alpha_k < 1 - {}^1/_2(n-1)\beta_1^2$$

The last inequality is identical with the condition of asymptotic n-stability of the rod in the quasistatic formulation of the problem /1/, where they where both necessary and sufficient.

Note that the conditions of stability of the rod as a dynamic system with one degree of freedom, acted upon by a force in the form of Gaussian white noise, were obtained in /5/. The application of the method of moment functions to the study of the stability of a rod acted upon by a stationary load with a rational-fraction spectral density was also discussed there.

3. ***Stability of a rod as a system with an infinite number of degrees of freedom.*** Let us consider a rod whose flexure at the initial instant is equal to

$$w(0, x) = \sum_{k=1}^{\infty} f_k^\circ \sin\frac{k\pi}{l}x$$

When the rod is excited by a load in the form of a stationary random process, the quasistatic formulation of the problem yields the following relation:

$$w(\tau, x) = \sum_{k=1}^{\infty} f_k^\circ \exp[-k^4(1-\alpha_k)\tau + k^2 z_1^*(\tau)]\sin\frac{k\pi}{l}x$$

When $\psi(\tau)$ is a Gaussian process, the following relation holds:

$$\langle\exp[\delta z_1^*(\tau)]\rangle = \exp[{}^1/_2\delta^2\langle z_1^*(\tau)\rangle]$$

Taking this relation into account, we can write the expression for the statistical n-th order moment of flexure of the rod in the form

$$\langle w^n(\tau, x)\rangle = \sum_{i_1=1}^{\infty}\cdots\sum_{i_n=1}^{\infty} f_{i_1}^\circ\cdots f_{i_n}^\circ \exp[\varkappa(\tau)]\sin\frac{i_1\pi}{l}x\cdots\sin\frac{i_n\pi}{l}x$$

$$\varkappa(\tau) = [-(i_1^4 + \ldots + i_n^4) + \delta\alpha_1]\tau + {}^1/_2\delta^2\langle z_1^{*2}(\tau)\rangle$$

$$\delta = i_1^2 + \ldots + i_n^2$$

Taking into account relation (1.11), we can write the condition of asymptotic stability of the rod with respect to the n-th order moments in the form

$$\alpha_1\delta < (i_1^4 + \ldots + i_n^4) - {}^1/_2(\mu a_i^{-1}\delta\beta_1)^2$$

Taking into account the relation

$$n\delta^{-2}(i_1^4 + \ldots + i_n^4) - 1 = \delta^{-2}[(i_1^2 - i_2^2)^2 + (i_1^2 - i_3^2)^2 + \ldots + (i_{n-1}^2 - i_n^2)^2]$$

we obtain

$$\alpha_1 < \frac{\delta}{n}\left\{\frac{1}{\delta^2}[(i_1^2 - i_2^2)^2 + \ldots + (i_{n-1}^2 - i_n^2)^2] + \left(1 - \frac{n\mu^2}{2a_i^2}\beta_1^2\right)\right\}$$

It is obvious that the relation obtained for the dimensionless value of the expectation of the compressive force $(\alpha_1 > 0)$ is satisfied the more, the more it is satisfied when $i_1 = \ldots = i_n = 1$:

$$\alpha_1 < 1 - {}^1/_2 n(\mu a_i^{-1}\beta_1)^2$$

It follows, therefore, that the rod (as a distributed system) is n-stable with respect to an arbitrary (deterministic) initial flexural perturbation, provided that the condition of n-stability of the rod with respect to the initial flexural perturbation, specified in the form of a single sinusoidal half-wave, is satisfied. We note that the argument also remains valid in the case when the amplitudes of the deflection of the rod at the initial instant are statistically independent of the load, and statistical n-th order moments exist for them.

The result obtained is of fundamental importance. We recall that the condition of stability of the rod compressed by a constant (deterministic) force follows from an analysis of its behaviour when the deflection is in the form of a single sinusoidal half-wave. The presence in the expansion of other sinusoids (with high order number) affects the magnitude of the deflection, but does not alter the condition for its stability. It is clear that in case of a stochastic formulation of the problem the situation is analogous.

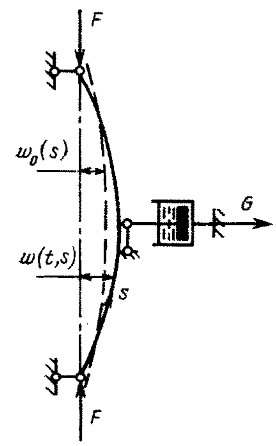


Fig.1

4. ***The stationary solution in the case of finite deflections of the rod.*** Let us consider a rod with initial deflection $w_0(s)$. In order to simplify the arguments which follow, we shall assume that a viscous coupling is applied at the middle of the rod, causing the reaction $G = -2w^{\cdot}(Al)$ (Fig.1). Assuming that the deflections of the rod are finite but fairly small, we can write the equation of slow motion

$$-EI\left\{\frac{\partial^2 w}{\partial s^2}\left[1+\frac{1}{2}\left(\frac{\partial w}{\partial s}\right)^2\right]-\frac{\partial^2 w_0}{\partial s^2}\left[1+\frac{1}{2}\left(\frac{\partial w_0}{\partial s}\right)^2\right]\right\}= \tag{4.1}$$

$$Fw-\frac{1}{Al}w^{\cdot}\int_0^s\left[1-\frac{1}{2}\left(\frac{\partial w}{\partial s}\right)^2\right]ds, \quad 0\leqslant s\leqslant\frac{l}{2}$$

When $l/2 \leqslant s \leqslant l$, an analogous expression is obtained from the condition that the rod is symmetrical.

Let us assume that $w_0(s) = f_0 \sin \pi l^{-1} s$. We shall seek $w(t, s)$ in the form

$$w(t, s) = f(t) \sin \pi l^{-1} s$$

After applying the Bubnov-Galerkin-Kantorovich-Vlasov procedure, we obtain from (4.1)

$$d\zeta/d\tau = -\nu[\zeta(1 + {}^1/_8\zeta^2) - \zeta_0(1 + {}^1/_8\zeta_0^2) - \alpha\zeta + \varkappa\zeta^2(\zeta - \zeta_0 - \alpha\zeta)];$$

$$\tau = \gamma t, \quad \zeta = \pi\frac{f}{l}, \quad \zeta_0 = \pi\frac{f_0}{l}, \quad \nu = \left(\frac{1}{4}+\frac{1}{\pi^2}\right)^{-1}, \quad \varkappa = \nu\left(\frac{1}{2\pi^2}+\frac{1}{16}\right)$$

We shall assume that $\alpha = \alpha_1 + \beta_1\xi(\tau)$; $\alpha_1, \beta_1 = \text{const}$, $\xi(\tau)$ is Gaussian, delta-correlated "white noise".

In what follows we shall restrict ourselves to considering only a stationary mode, for which the FPK equation will take the form ($\partial v/\partial\tau = 0$)

$$\frac{\partial}{\partial\zeta}(av) + \frac{1}{2}\frac{\partial^2}{\partial\zeta^2}(b^2 v) = 0$$

and

$$a = \mu\{[(1-\alpha_1)\zeta - \zeta_0](1 + \varkappa\zeta^2) + {}^1/_8(\zeta^3 - \zeta_0^3)\}, \quad b = \mu\beta_1\zeta(1 + \varkappa\zeta^2)$$

The amplitude distribution density of the deflection of the rod is given by the expression

$$v(\zeta) = c\zeta^{-\rho}(1 + \varkappa\zeta^2)^{\rho/2-3}\exp\left(-\frac{2}{\mu\beta_1^2}\varepsilon\right)$$

$$\varepsilon = \frac{\zeta_0}{\zeta}\left(1 + \frac{\zeta_0^2}{8}\right) - \frac{1 - \varkappa^2\zeta_0^3\zeta}{16\varkappa(1 + \varkappa\zeta^2)} + \sqrt{\varkappa}\,\zeta_0\left(1 + \frac{3}{16}\zeta_0^2\right)\operatorname{arctg}\sqrt{\varkappa}\,\zeta,$$

$$\rho = 2\left(1 + \frac{1-\alpha_1}{\mu\beta_1^2}\right), \quad \left(\int_0^\infty v(\zeta)\,d\zeta = 1\right)$$

The constant c is found from the normalization condition, given above in the brackets. The statistical n-th order moment of the quantity ζ is equal to

$$\langle\zeta^n\rangle = c\int_0^\infty \zeta^{n-\rho}(1 + \varkappa\zeta^2)^{\rho/2-3}\exp\left(-\frac{2\varepsilon}{\mu\beta_1^2}\right)d\zeta \tag{4.2}$$

Let us assume that $\zeta_0 = 0$. Then we have

$$\frac{1}{c} = \int_0^\infty \zeta^{-\rho}(1+\varkappa\zeta^2)^{\rho/2-3}\exp\left[-\frac{1}{8\mu\beta_1^2\varkappa(1+\varkappa\zeta^2)}\right]d\zeta$$

The necessary condition for the existence of the distribution density $v(\zeta)$ and statistical moments is that the following conditions hold:

$$\alpha_1 > 1 + {}^1/_2\,\mu\beta_1^2,\ \alpha_1 > 1 - {}^1/_2\,(n-1)\,\mu\beta_1^2\,(n<5) \tag{4.3}$$

Clearly, when the first relation holds, so does the second.

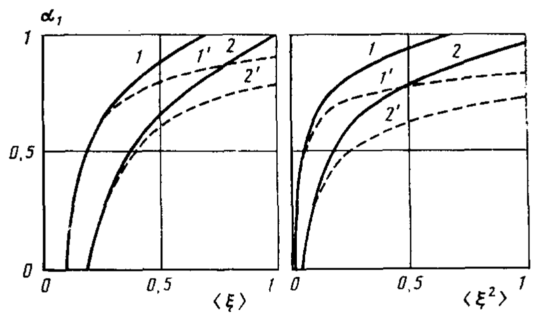
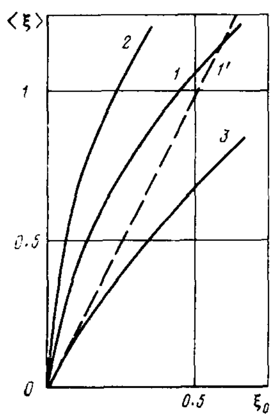


Fig.2



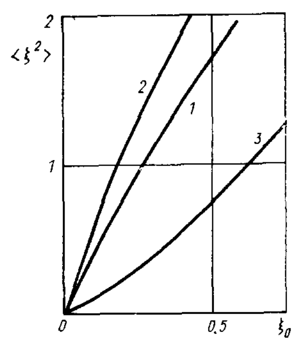


Fig.3 Fig.4

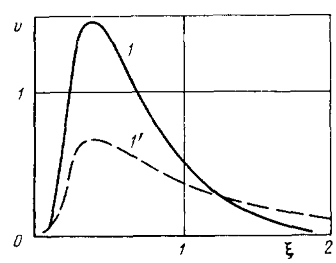


Fig.5

By considering the linear problem we can show that

$$\alpha_1 < 1 - {}^1/_2\,(n-1)\,\mu\beta_1^2 \tag{4.4}$$

represents the condition of asymptotic n-stability of the motion of a straight rod. We see that the inequalities (4.3) are opposite to the inequality (4.4). This implies that when the conditions of asymptotic stability of the motion of the rod are violated, a stationary mode may appear. The existence of amplitude distribution density of the stationary mode represents the criterion for its appearance.

here we observe the analogy between the results of solving the stochastic and the deterministic problem.

Indeed, when a straight rod is compressed by a constant (deterministic) force smaller than the Euler force $(\alpha_1 < 1)$, we have only a single form of equilibrium which is rectilinear and stable. On the other hand, if the force is greater than the Euler force $(\alpha_1 > 1)$, then in addition to the rectilinear equilibrium we have another, - the flexural form of the equilibrium of the rod, and the first form is unstable, while the other form is stable.

Analysis of expression (4.2) shows that when $\zeta_0 \neq 0$, the moment $\langle\zeta^n\rangle$ exists for any α_1, β_1 when $n < 5$. We note that in the case of small deflections the statistical n-th order moment exists only when conditions (4.4) hold.

Here it is again relevant to note the analogy between the solutions of the deterministic and stochastic problems. When we consider the linear deterministic problem of the longitudinal flexure of the rod whose axis is curved at the beginning, a solution will exist if the compressive force is smaller than the Euler force $(\alpha_1 < 1)$. If, on the other hand, we take into account the finite deflections of the rod, then a solution of the problem will exist for any value of the compressive force.

It is interesting to compare the results of solving the same problem in the linear and non-linear formulation. Fig.2 shows graphs of the variation in the first and second moment of the quantity ζ as a function of α_1, for $\beta_1 = 0.5$. Curves *1* and *2* are constructed from the results of solving the non-linear problem for $\zeta_0 = 0.1$ and 0.2. Curves *1'* and *2'* correspond to the solution of the linear problem. Figs.3 and 4 show graphs of the variation of the first and second moment of the quantity ζ as a function of ζ_0. Curves *1-3* correspond to the parameters $\alpha_1 = 1,\ \beta_1 = 1,\ \alpha_1 = 1,\ \beta_1 = 0.5;\ \alpha_1 = 0.5$ and $\beta_1 = 1$. When the linear formulation is used, the above moments do not exist except for the single case $\langle\zeta\rangle \sim \zeta_0$ when $\alpha_1 = 0.5$ and $\beta_1 = 1$, which is depicted in Fig.3 by line *1'*.Fig.5 shows graphs of the variation of the stationary density distribution of the probabilities for $\alpha_1 = 1, \beta_1 = 0.5$ and $\zeta_0 = 0.1$. Curve *1* corresponds to the solution of the non-linear problem, and curve *1'* to that of the linear problem.

The above results show that taking into account the finite deflections leads not only to a qualitative, but also to a quantitative difference between the results compared with the solution of the problem in which the deflections are assumed to be small.

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